# Cooperative Data Exchange with Weighted Cost based on d-Basis Construction

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# Outline

- Cooperative Data Exchange
  - Problem Setup
  - Example and Existing methods
- Main Results
  - A deterministic algorithm to compute the minimum number of required transmissions
  - Optimal coding schemes in which each transmission is a linear combination of fixed number of packets
  - An efficient way to generate coefficient matrix of linear coding scheme starting from Vandermonde matrix

# Problem Setup

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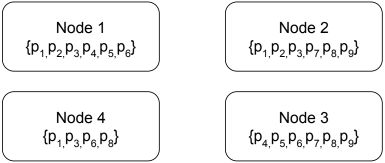
- A $K$ packet making up file.
- Each node initially has a subset of the $K$ packets and knows the packet distributions of other nodes.
- Goal: All node recovers all packets (Universal Recovery).
- Question:
  - What is the minimum number of required transmissions?
  - How to construct the optimal coding scheme?

# Example

## 4 Nodes and 9 Packets



# Example

## 4 Nodes and 9 Packets

Node 1
$\{p_1, p_2, p_3, p_4, p_5, p_6\}$

Node 2
$\{p_1, p_2, p_3, p_7, p_8, p_9\}$

Node 4
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Node 3
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$$E = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

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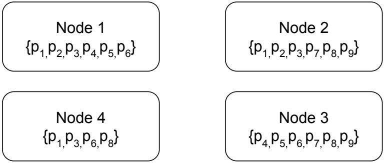


- Minimum number of required transmissions

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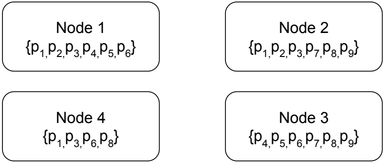


- Minimum number of required transmissions
  - $R^* = 5$

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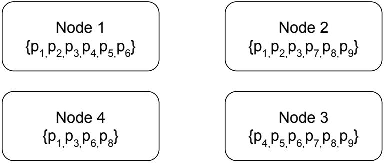


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  $T_1 = p_1 + p_5$,
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  Unique?

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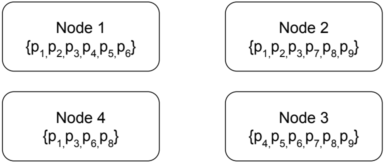


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  $T_1 = 5p_1 + 4p_2 + 4p_3 + p_4 + p_5,$
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  $T_4 = 9p_1 + 12p_2 + 7p_3 + 15p_7 + 14p_9,$
  $T_5 = 10p_4 + 14p_5 + 6p_6 + 9p_7 + 8p_8$
  (over $GF(2^4)$ with primitive polynomial $\alpha^4 + \alpha + 1$)

# Problem Formulation

## Integer linear program with Slepian-Wolf Constraints on all proper subsets

The cooperative data exchange problem can be formulated as the following **Integer Linear Program**:

$$\text{minimize} \sum_{i=1}^{N} r_i$$

$$\text{subject to} \sum_{i \in \mathbf{S}} r_i \geq \left| \bigcap_{i \in \mathbf{S}^c} X_i^c \right|, \forall \emptyset \subsetneq \mathbf{S} \subsetneq [N]$$

$X_i$ : The set of packets that are available at node $i$.
$r_i$ : The number of transmissions sent by node $i$.

# Background

## d-Basis Construction

[Li et al.'17] proved, for the basic CDE problem:

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[Li et al.'17] proved, for the basic CDE problem:

- The existence of d-Basis is the sufficient and necessary condition for achieving Universal Recovery with $K - d$ transmissions.
- We can always generate an optimal linear coding scheme in which each transmission is a linear combination of d + 1 packets and those packets are indexed by d-Basis vectors.
- The coefficient matrix can be efficiently generated by performing elementary row operations on a Vandermonde matrix.

# Definitions

## Definition: $d$-Basis

A set of $K$-dimensional binary vectors ($\mathbf{V} = \{v_i : i \in [K-d]\}$) is called a $d$-Basis if

$$\begin{aligned} w_H(v_i) &= d+1, && \forall v_i \in \mathbf{V} \\ w_H(v_{\mathbf{S}}) &\geq |\mathbf{S}| + d, && \forall \emptyset \subsetneq \mathbf{S} \subsetneq \mathbf{V} \end{aligned}$$

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$v_1 = [1\ 1\ 1\ 0\ 0]$, $v_2 = [1\ 1\ 0\ 1\ 0]$,
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$\mathbf{F} = \{v_1, v_2, v_3\}$, $w_H(v_{\mathbf{F}}) = 4$ $\rightarrow$ not vectors of 2-Basis

# Definitions

## Definition: Vector Production

A binary vector $u$ can generate another binary vector $v$ if $u$ and $v$ have the same dimensions and $supp(v) \subseteq supp(u)$.
Let $\mathcal{G}(u, d)$ denote set of all binary vectors that can be generated by $u$ and have $d+1$ ones. $\mathcal{G}(\mathbf{S}, d) = \cup_{u \in \mathbf{S}} \mathcal{G}(u, d)$.

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## Example

$e_1 = [1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0]$ can generate the following two 4-Basis vectors:

$$v_1 = [1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0] \qquad v_2 = [1\ 1\ 1\ 1\ 0\ 1\ 0\ 0\ 0]$$

# Minimum Number of Required Transmissions



## Theorem 1

If for some subset of nodes $\mathbf{I} \subseteq \mathbf{N}$ there exists a $d$-Basis $\mathbf{V} \subseteq \mathcal{G}(\{e_i, i \in \mathbf{I}\}, d)$, then the nodes of $\mathbf{I}$ can generate a coding scheme $\mathbf{T} = \{T_1, \ldots, T_R\}$ with $R = K - d$ such that $\forall i \in \mathbf{N}, w_H(e_i) \geq d$, node $i$ can recover all packets.

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In $\mathcal{G}(E, 4)$, we can find a 4-Basis as

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

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There exists a coding scheme with 5 transmissions in which each transmission is a linear combination of 5 packets. Nodes with at least 4 packets can recover all packets.

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necessity of coding scheme based on $d$-Basis

**Theorem 2**

If universal recovery can be achieved by a linear coding scheme with $R$ $(R = K - d)$ transmissions, then the PDVs of the nodes can generate a $d$-Basis $\mathbf{V} = \{v_1 \ldots, v_R\}$.

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As we know a coding scheme with 5 that can achieve universal recovery, the PDVs of nodes can generate a 4-Basis.

**Corollary**

If the PDVs of nodes cannot generate any $d$-Basis, then there does not exist any linear coding scheme with $K - d$ transmissions that can achieve universal recovery.

# Minimum Number of Required Transmissions

**Theorem 3**

For the CDE in the fully connected network, the minimal number of required transmissions $R^*$ satisfies:

$$R^* = K - \min\{\mathcal{M}, d^*\} \quad (1)$$

where the $d^*$-Basis is the largest $d$-Basis that can be generated by the PDVs and $\mathcal{M} = \min_{i \in \mathbf{N}} |X_i|$ is the minimal number of initially available packets at any single node.

# Find $d^*$

## Polynomial-time Deterministic Algorithm

### Algorithm 1

For a given $d$, determine whether any $d$-Basis can be generated or not.

# Find $d^*$

## Polynomial-time Deterministic Algorithm

### Algorithm 2

Find the maximum value of $d$ such that $d$-Basis can be generated by binary search method.

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The overall complexity is bounded by $\mathcal{O}(N^3 K^3 \log(K))$,

- Only search for existence of coding schemes based on $d$-Basis
- $d$-Basis vectors are mergeable

# Optimal Linear Coding Scheme

## Based on $d$-Basis

The $d$-Basis specifies the packets that should be used to generate each transmission.

$$v_1 = [1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0]$$

# Optimal Linear Coding Scheme

## Based on $d$-Basis

The $d$-Basis specifies the packets that should be used to generate each transmission.

$$v_1 = [1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0]$$

But the each real transmission is a linear combinations of such packets with coefficient vector:

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & a_{26} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 0 & a_{27} & a_{28} & 0 \\ a_{41} & a_{42} & a_{43} & 0 & 0 & 0 & a_{27} & 0 & a_{28} \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} & a_{57} & a_{58} & 0 \end{bmatrix}$$

# Optimal Linear Coding Scheme

## Coefficient matrix is from MDS Codes

Vandermonde matrix $\mathcal{V}$ with $R$ rows and $K$ columns

$$\mathcal{V} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 & 1 \\ \theta_1 & \theta_2 & \theta_3 & \dots & \theta_{K-1} & \theta_K \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \theta_1^{R-1} & \theta_2^{R-1} & \theta_3^{R-1} & \dots & \theta_{K-1}^{R-1} & \theta_K^{R-1} \end{bmatrix}$$

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over a large enough finite field ($GF(2^4)$with primitive polynomial $\alpha^4 + \alpha + 1$) and $\theta_i = i$.

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$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 4 & 5 & 3 & 2 & 7 & 6 & 12 & 13 \\ 1 & 8 & 15 & 12 & 10 & 1 & 1 & 10 & 15 \\ 1 & 3 & 2 & 5 & 4 & 6 & 7 & 15 & 14 \end{bmatrix}$$

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By elementary row operations:

$$A = \begin{bmatrix} 5 & 4 & 4 & 1 & 1 & 0 & 0 & 0 & 0 \\ 15 & 11 & 14 & 14 & 0 & 1 & 0 & 0 & 0 \\ 3 & 6 & 13 & 0 & 0 & 0 & 15 & 14 & 0 \\ 9 & 12 & 7 & 0 & 0 & 0 & 15 & 0 & 14 \\ 0 & 0 & 0 & 10 & 14 & 6 & 9 & 8 & 0 \end{bmatrix}$$

# Summary

## Contributions

- We present a new deterministic algorithm to compute the minimum number of required transmissions. The complexity of our algorithm is bounded by $\mathcal{O}(N^3 K^3 \log(K))$.

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- We present a new deterministic algorithm to compute the minimum number of required transmissions. The complexity of our algorithm is bounded by $\mathcal{O}(N^3K^3 \log(K))$.
- We propose a novel coding scheme with $K - d$ transmissions in which each transmission is a linear combination of $d + 1$ packets.
- The coefficient matrix of our coding scheme can be efficiently generated by performing elementary row operations on a Vandermonde matrix.

Thank you!