On Learning Continuous Pairwise Markov Random Fields

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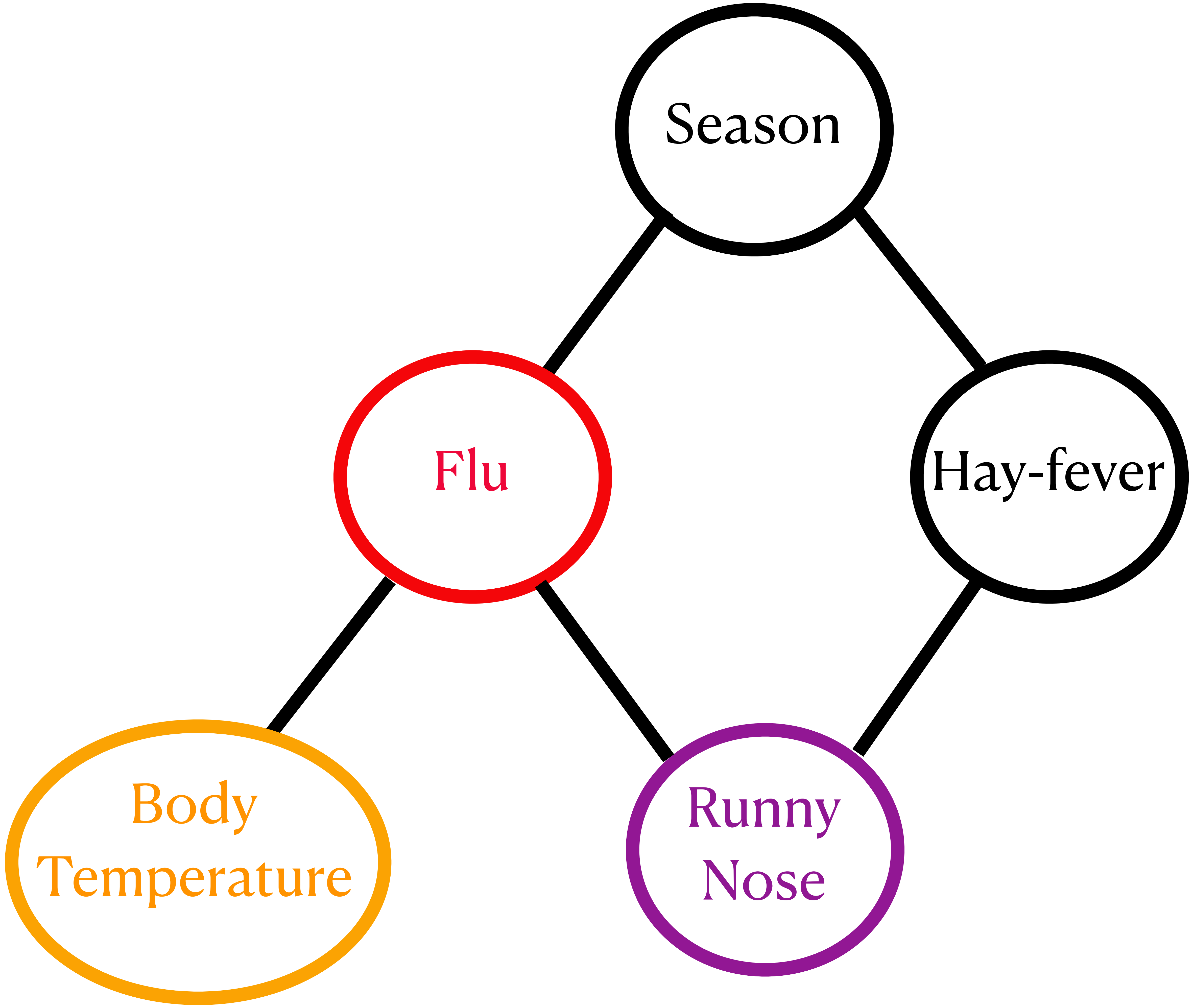
Markov Random Fields

Undirected Graphical Models

- Diagrammatic representations of probability distributions with a Markovian structure

Local Markov Property

- Given the value of neighbors, a node is independent of the remaining nodes
- Body Temperature $\perp\!\!\!\perp$ Runny Nose | Flu



Pairwise Markov Random Fields

- Consider an undirected graph $G = ([p], E)$

$$f_{\mathbf{x}}(\mathbf{x}) \propto \exp\left(\sum_{i \in [p]} g_i(x_i) + \sum_{(i,j) \in E} g_{ij}(x_i, x_j)\right)$$

- Examples — Ising model, discrete graphical model, Gaussian graphical model
- Limited progress for continuous (non-Gaussian) MRFs

Algorithm

Overview

1. Recover the graph structure and the associated edge parameters —
   1.1. Extend the Generalized Interaction Screening Objective (GISO) to the continuous setting
   1.2. If $f_{\mathsf{X}_i}(x_i|\mathsf{X}_{-i} = x_{-i}) \propto \exp\left(g(\boldsymbol{\theta}, \mathbf{x})\right)$, then GISO = $\mathbb{E}\left[\exp\left(-g(\boldsymbol{\theta}, \mathbf{x})\right)\right]$.
2. Recover the node parameters —
   2.1. Transform the problem of learning node parameters to a sparse linear regression
   2.2. Use a robust variation of lasso, and knowledge of the learned edge parameters

Main Results

Finite-sample guarantees

- Structure recovery and parameter recovery with $\Omega(\log(p))$ samples.
- Do not require abstract conditions such as the incoherence, dependency, sparse eigenvalue or restricted strong convexity.

Main Results

Understanding GISO

- Minimizing the population version of GISO is equivalent to a ‘local’ MLE.
- Under mild conditions, the finite sample estimate of GISO is asymptotically consistent and normal.

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- Even though the traditional MLE is intractable, this ‘local’ M-estimation is tractable.
- However, unlike traditional MLE, this is not asymptotically efficient.

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