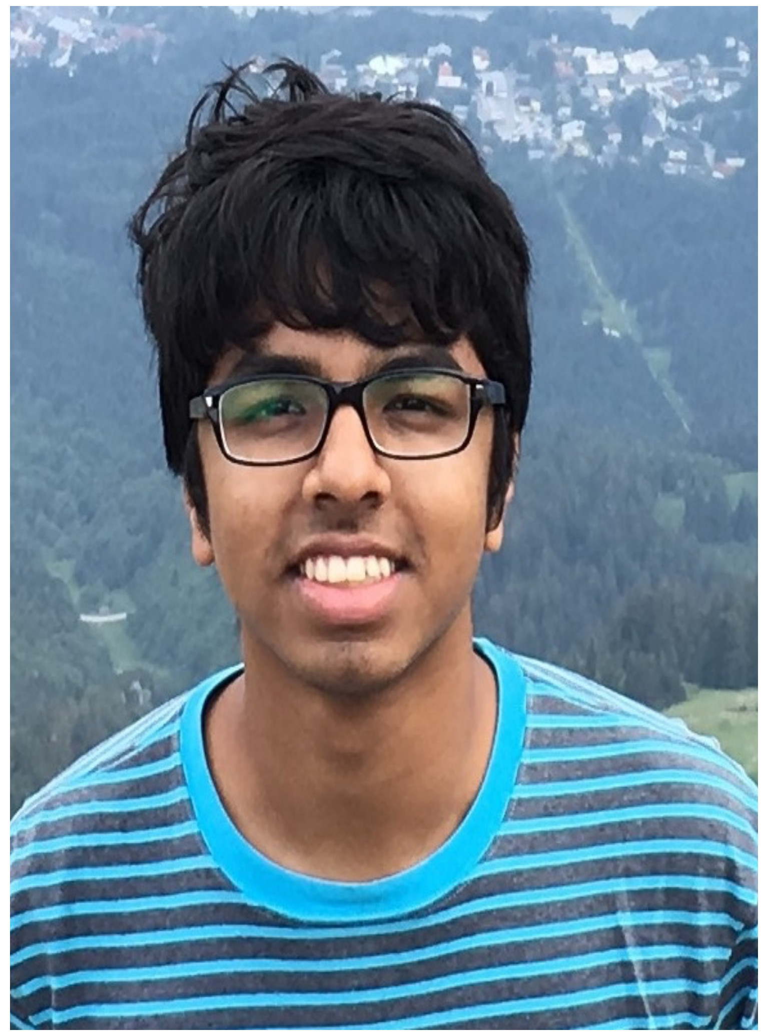
Finding Valid Adjustments under Non-ignorability with Minimal DAG Knowledge



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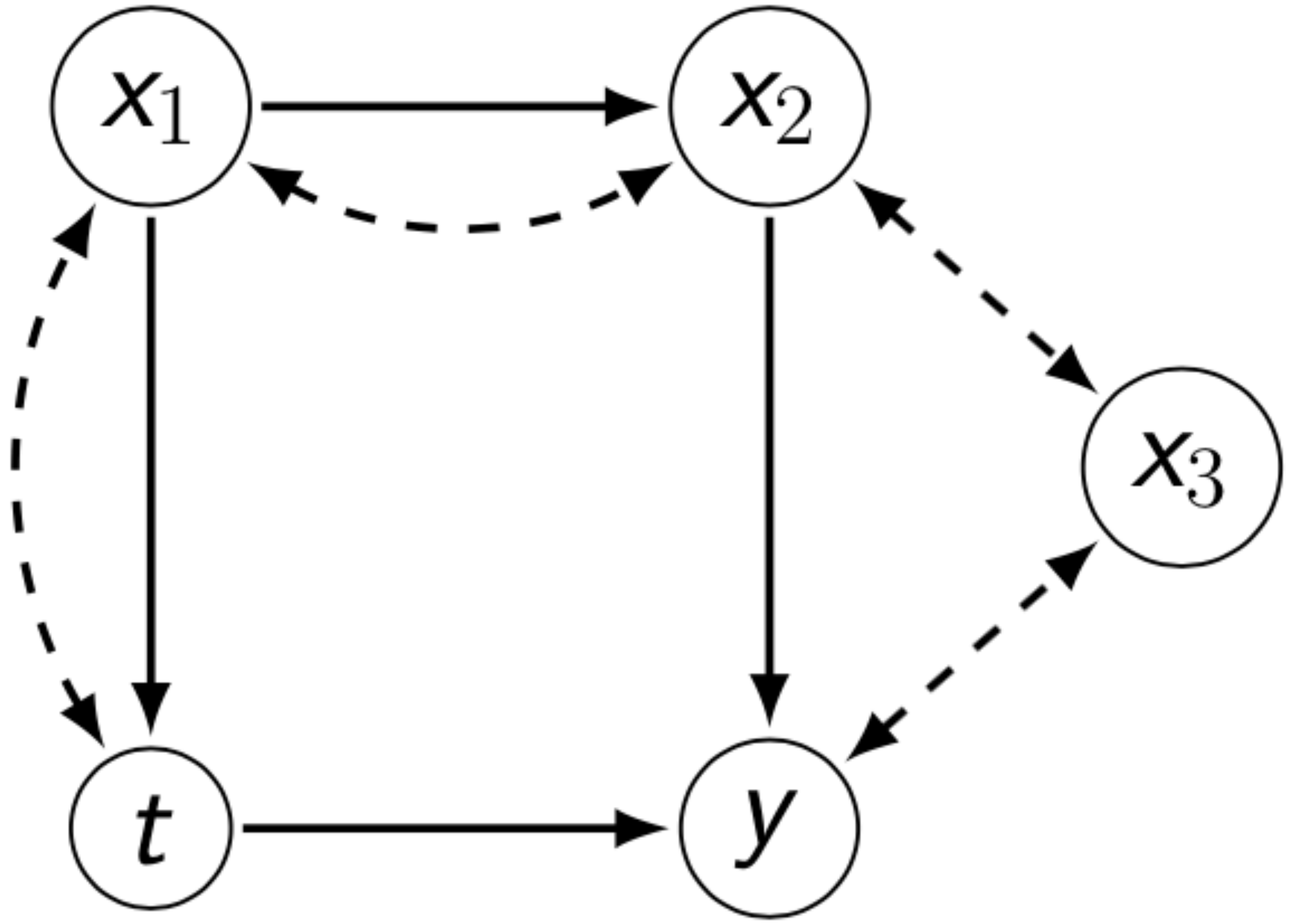
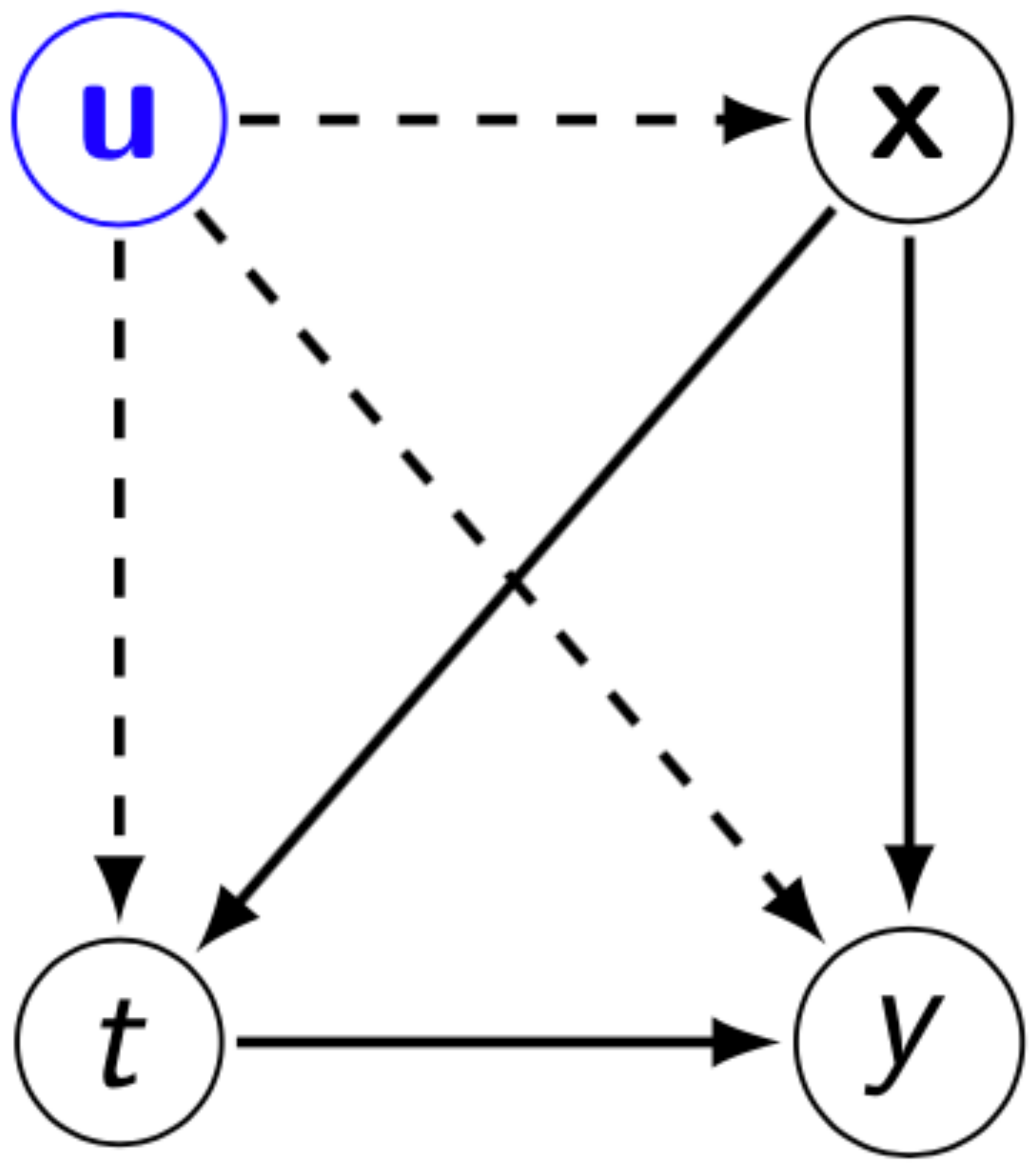
How much of the DAG do we need to know?

To find the causal effect of t on y i.e., $\mathbb{P}(y \mid do(t = t))$ from observational data



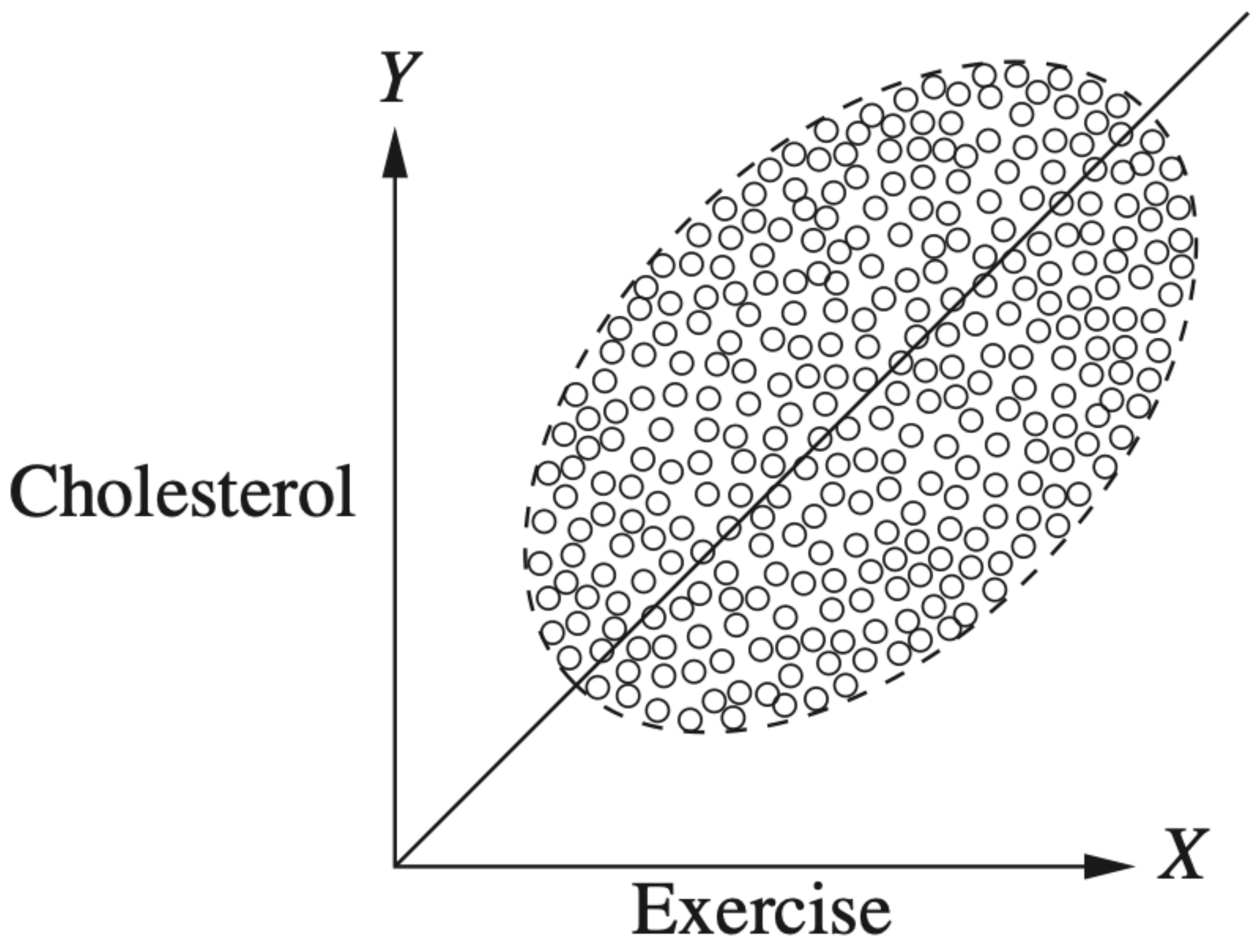
Causal Effect Estimation

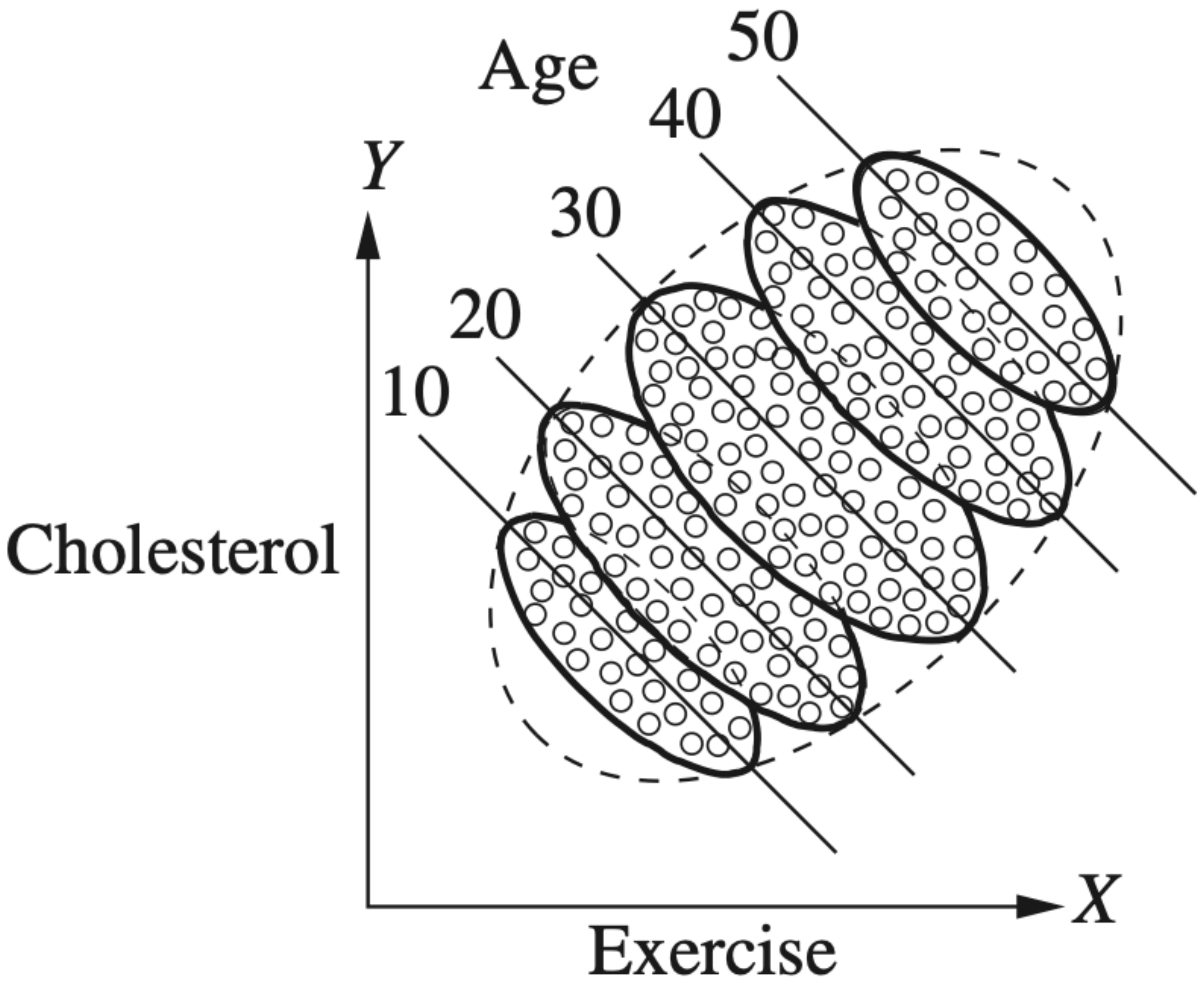
- $\mathbf{u}$: unobserved exogenous variables
- $\mathbf{x}$: observed features
- t : observed binary treatment variables
- y : observed outcome
- $\mathcal{G}$: DAG over the set of vertices $\{\mathbf{u}, \mathbf{x}, t, y\}$



Valid adjustments

Simpson's paradox: Which subsets of the observed features should be adjusted for?





More exercise $\Longrightarrow$ more cholesterol

More exercise $\Longrightarrow$ less cholesterol when adjusted for age group

z is a valid adjustment set if $\mathbb{P}(y \mid do(t = t)) = \mathbb{E}_{\mathbf{z}}[\mathbb{P}(y \mid \mathbf{z} = z, t = t))]$

Pearlian Framework

DAG knowledge

Given the complete knowledge of the DAG, different graphical criteria (e.g., back-door criterion, front-door criterion) could be used to check whether a subset is valid for adjustment

Potential Outcomes

Ignorability

$\mathbf{z}$ satisfies ignorability if $y_0, y_1 \perp t \,|\, \mathbf{z}$

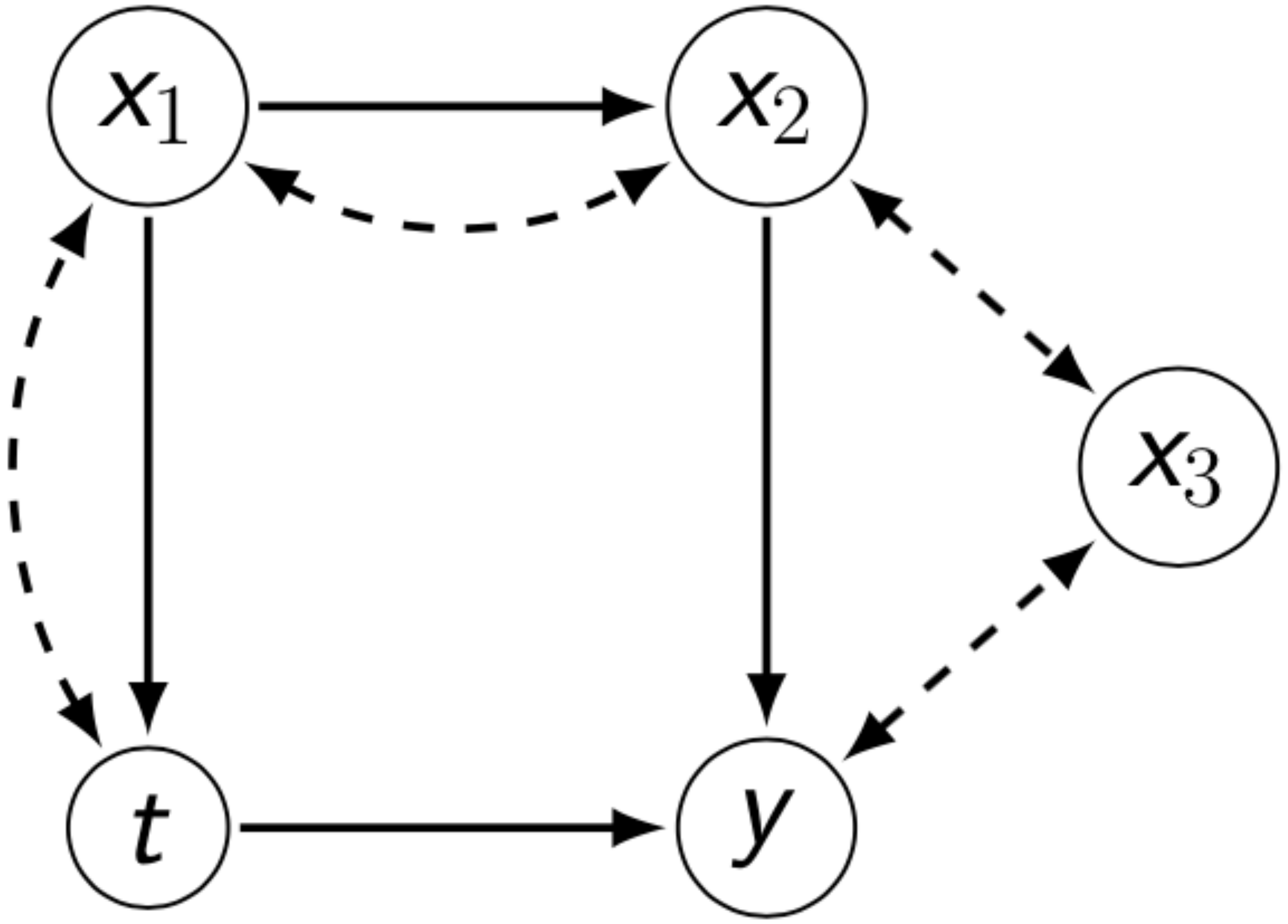
Ignorability implies that $\mathbf{z}$ is a valid adjustment.

z is a valid adjustment set if $\mathbb{P}(y \,|\, do(t = t)) = \mathbb{E}_{\mathbf{z}}[\mathbb{P}(y \,|\, \mathbf{z} = z, t = t))]$

Assumptions

Semi-Markovian model

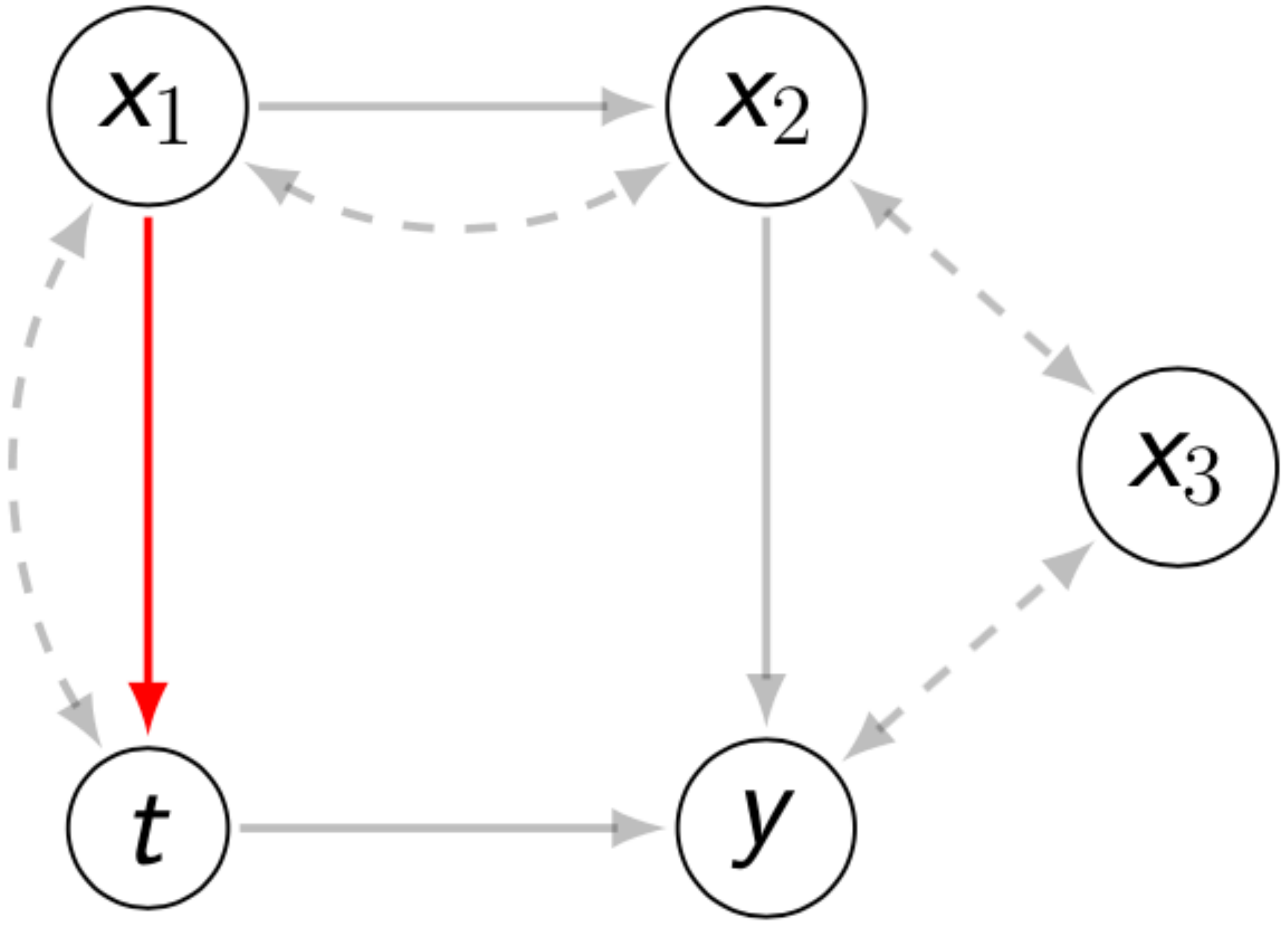
1. Let the DAG $\mathcal{G}$ be such that the treatment t has the outcome y as its only child.
2. Let the DAG $\mathcal{G}$ be such that the outcome y has no child.



We do not assume ignorability

How much of the DAG do we need to know?

Can we significantly reduce the structural knowledge required about the DAG and yet find valid adjustment sets under non-ignorability?



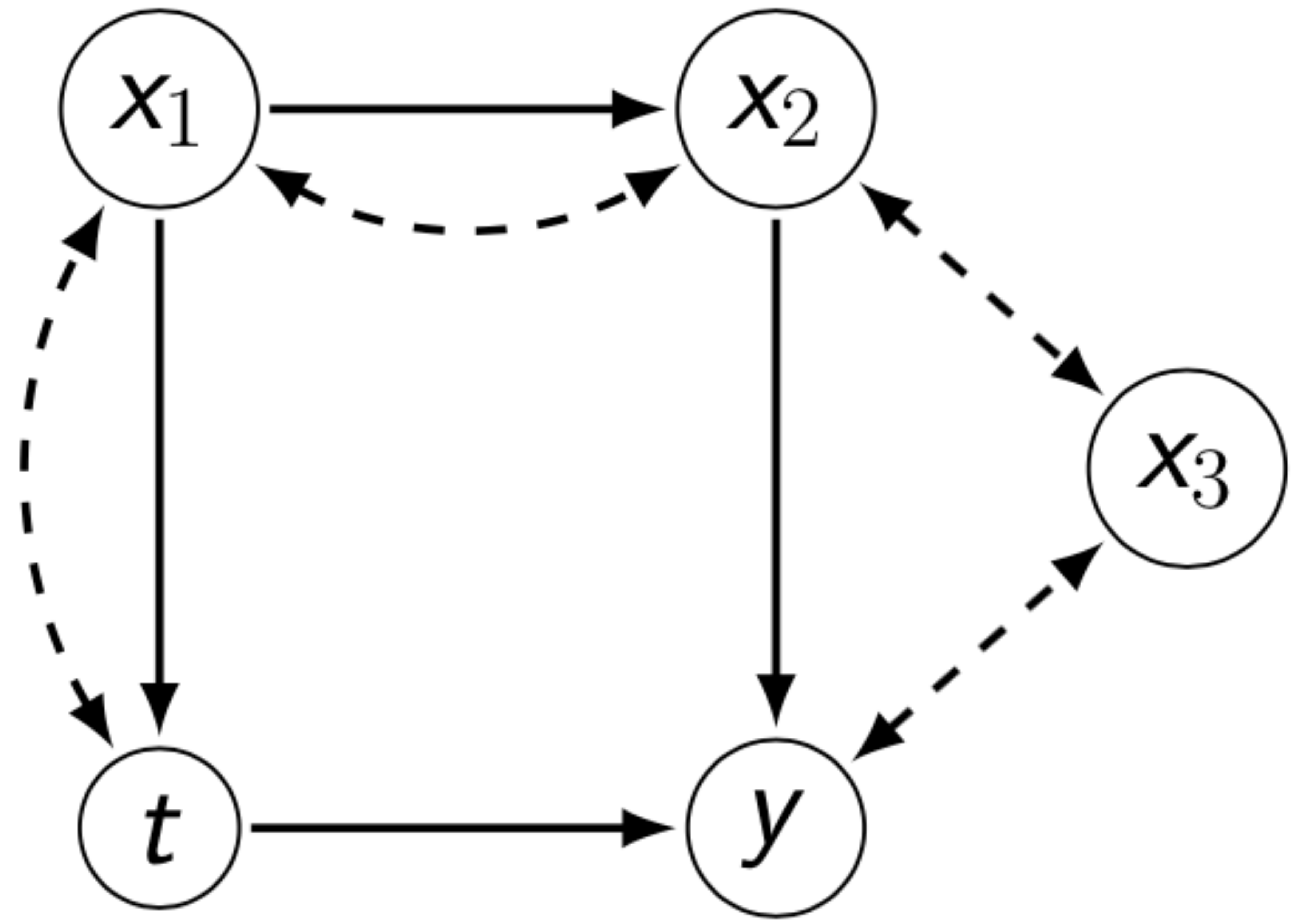
The knowledge of one causal parent of the treatment is sufficient to find a class of valid adjustment sets!

Back-door Criterion

A popular sufficient graphical criterion for finding valid adjustments

Under our assumptions, a set $\mathbf{z}$ satisfies the back-door criterion in $\mathcal{G}$ if

1. $\mathbf{z}$ blocks every path between t and y in $\mathcal{G}$ that contains an arrow into t.



Sets satisfying back-door:

- $\{x_1, x_2\}$
- $\{x_2\}$

Equivalence

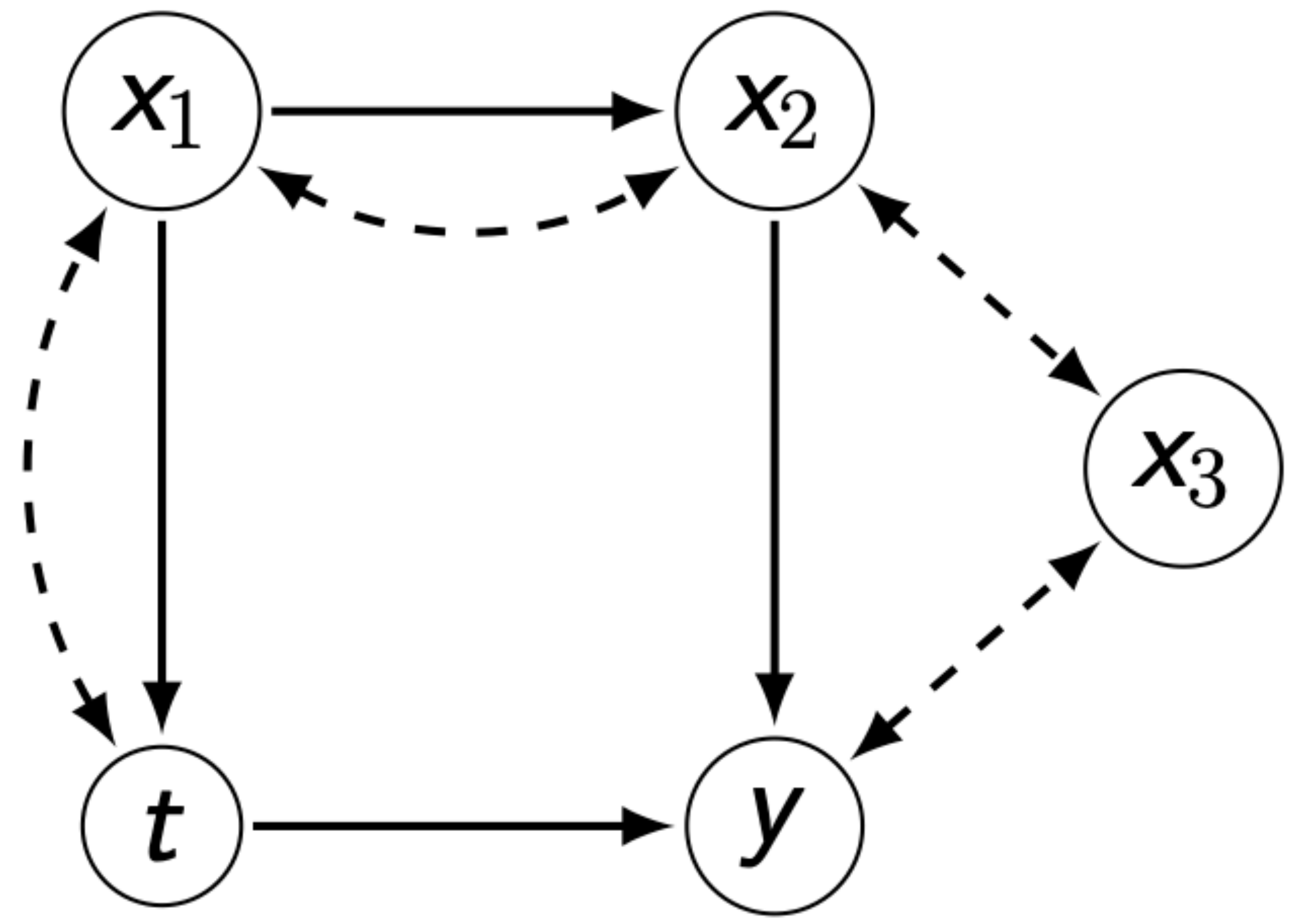
invariance testing $\Longleftrightarrow$ back-door criterion*

- x_t : an observed feature that is a direct causal parent of t.
- Consider any subset of the remaining observed features i.e., $\mathbf{z} \subseteq \mathbf{x} \setminus \{x_t\}$.
- $\mathbf{z}$ satisfies the back-door criterion if and only if $x_t \perp y \mid \mathbf{z}, t$.

*Forward direction is implied by Entner et al. (2013)

An illustrative example.

$x_t = x_1$



- $\mathbf{z} \in \{\emptyset, \{x_2\}, \{x_3\}, \{x_2, x_3\}\}$
- Only $\mathbf{z} = \{x_2\}$ satisfies the back-door criterion
- Only $\mathbf{z} = \{x_2\}$ satisfies $x_1 \perp_d y \,|\, z, t$

Algorithms

- Subset Search:

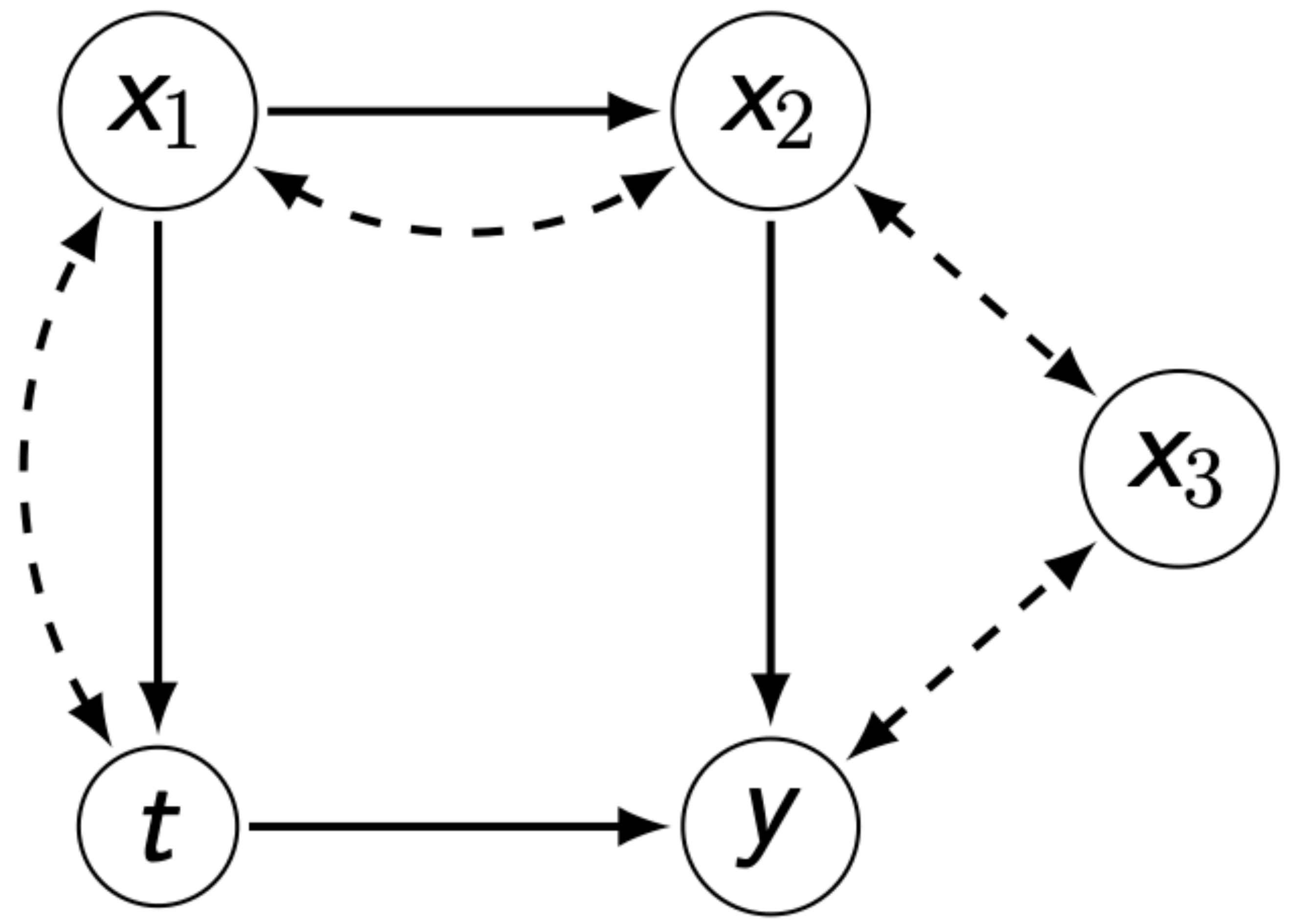
  1. We use a subset based search procedure that exploits conditional independence (CI) testing to check our invariance criterion.

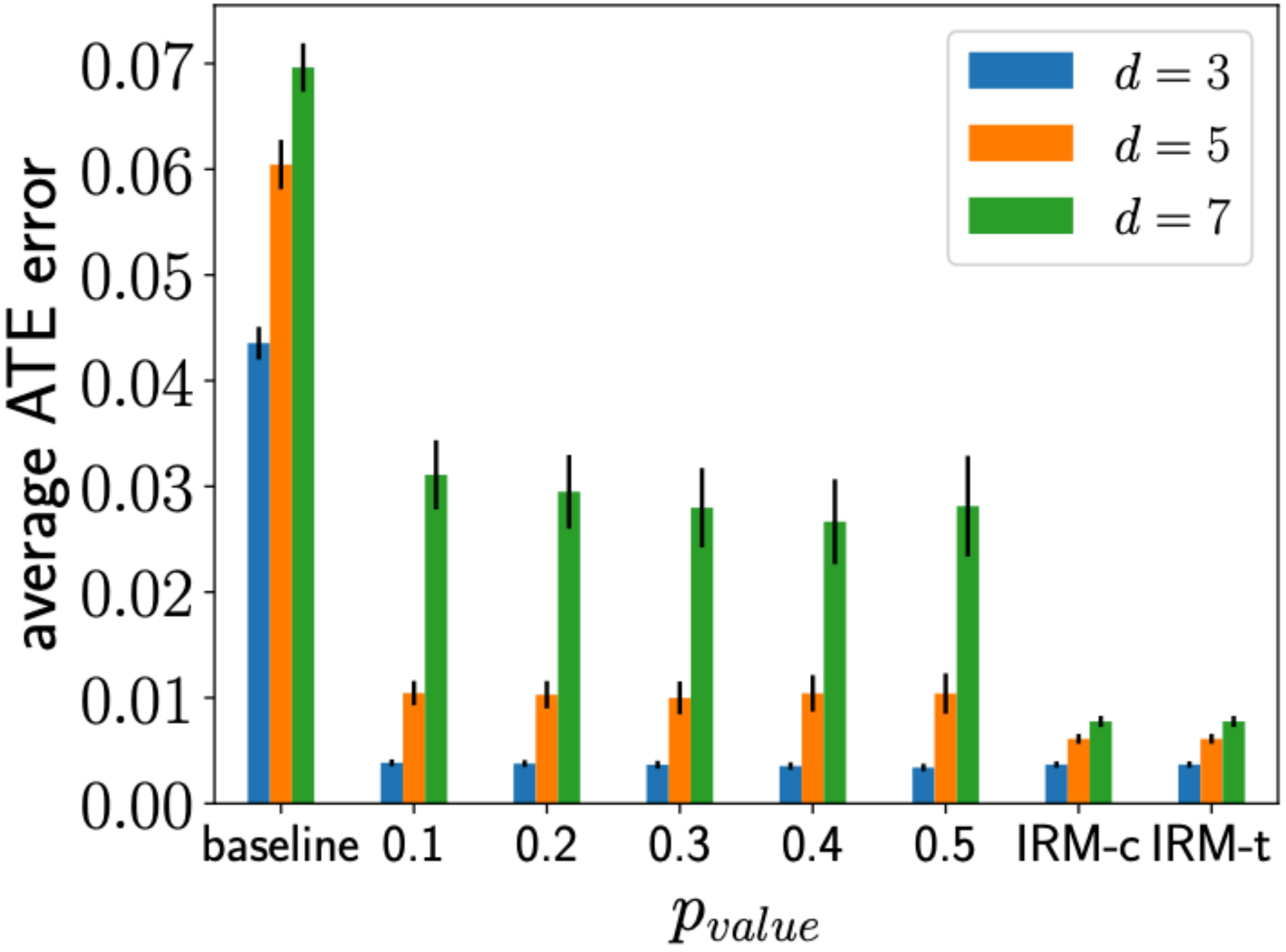
  p_{value} **: check if the p-value returned by the CI tester is more than** p_{value}

- IRM-based:

  1. We use a sub-sampling trick to leverage Invariant Risk Minimization (IRM) (Arjovsky et al., 2019) as a scalable approximation for CI testing.

Synthetic Experiment



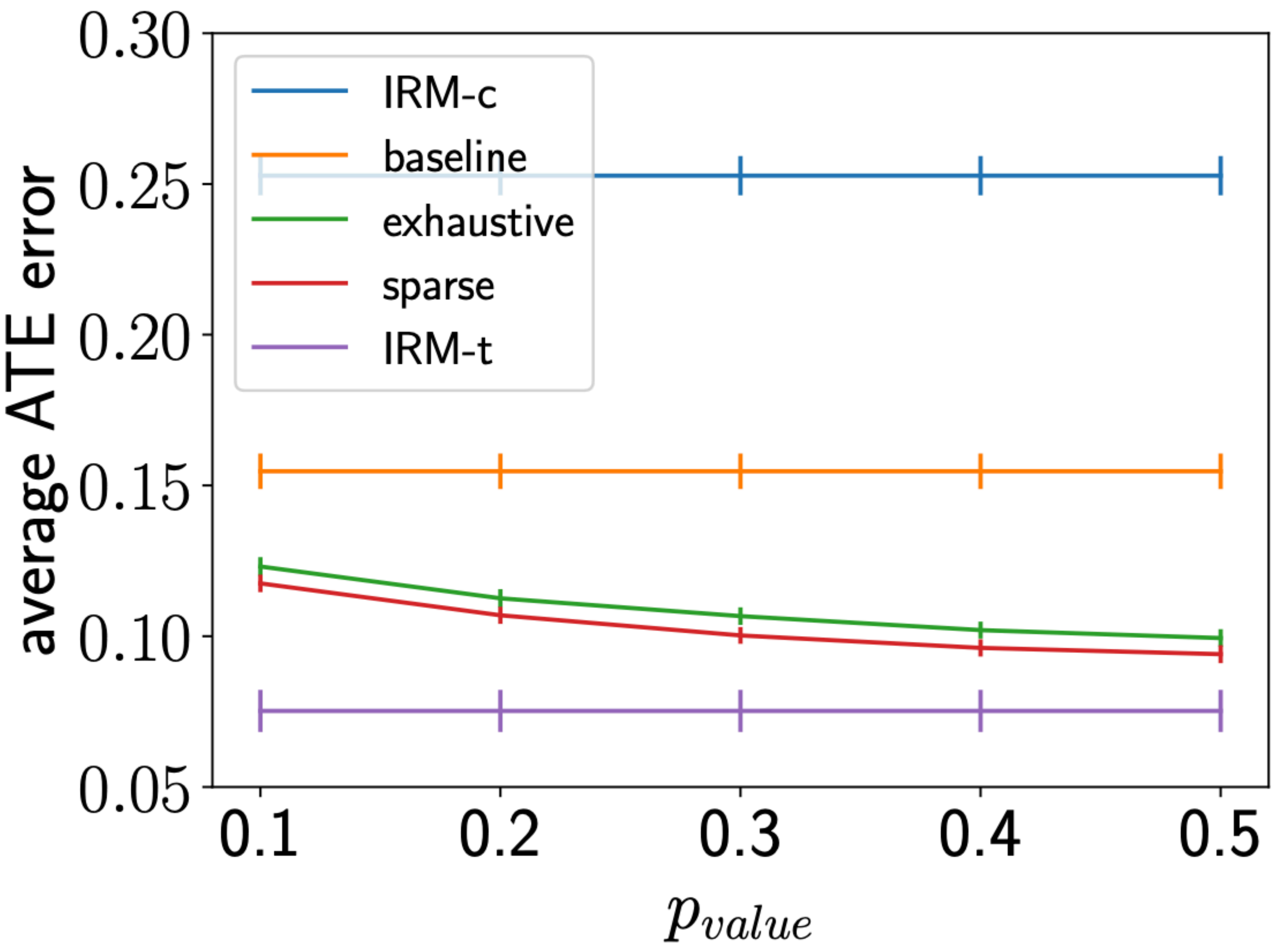


Performance of our algorithm with $x_t = x_1$

IRM-based approach can scale well in high dimensions ($d = 65$)!

IHDP

A RCT studying cognitive test score of low-birth-weight, premature infants.



Thank you!