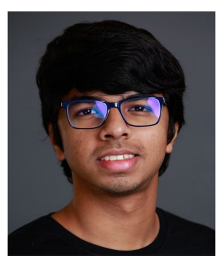
Front-door Adjustment Beyond Markov Equivalence with Limited Graph Knowledge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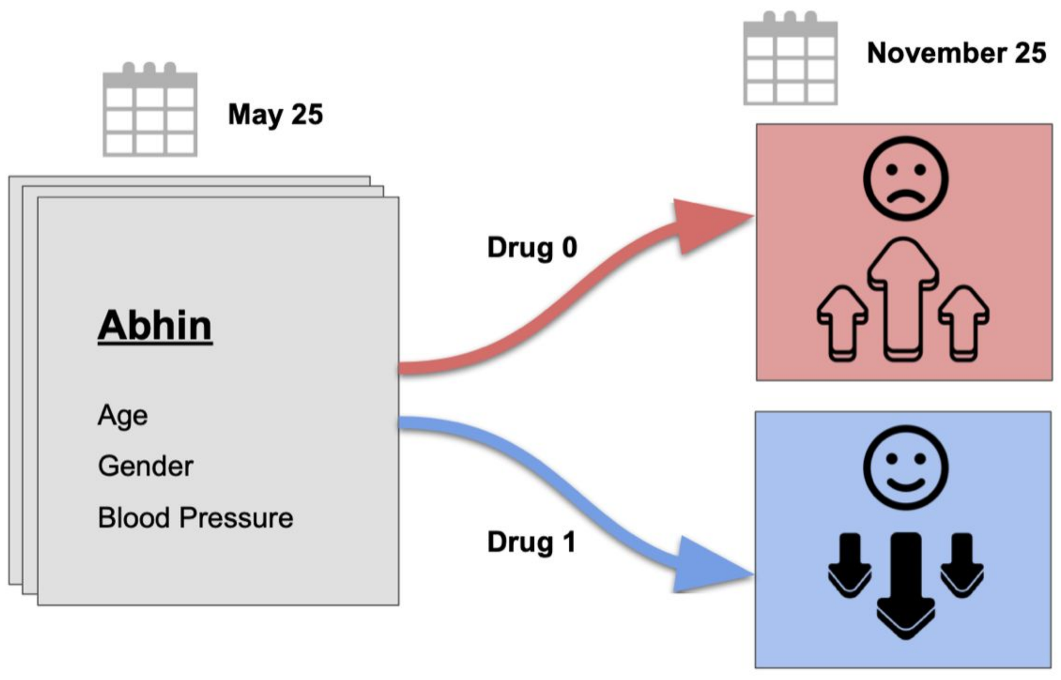
arXiv Link

Causal Effect Estimation

Causal effect of a drug on cholesterol level from observational data



$\mathbb{P}(\text{cholesterol} \mid do(\text{drug}))$?

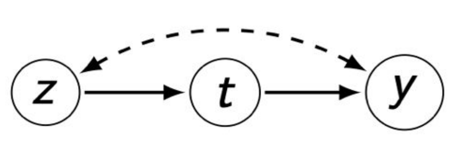
Observational Data

Age	Gender	Blood Pressure	Drug	Cholesterol (0)	Cholesterol (1)
22	Male	145/95	0	↑↑↑	?
26	Female	135/80	0	↓↓↓	?
58	Female	130/70	1	?	↑↑↑
50	Male	145/80	1	?	↓↓↓
24	Female	150/85	1	?	↑↑↑

Observed features $\mathbf{x}$ — Treatment t — Outcome y

Causal Estimands with known graph

Back-door Criterion

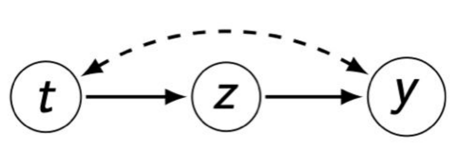


A set $\mathbf{z}$ satisfies the back-door criterion if

1. all back-door paths between t and y are blocked by $\mathbf{z}$ (i.e., paths containing an arrow into t)
2. $\mathbf{z}$ does not contain any descendant of t

$$\mathbb{P}(y \mid do(t=t)) = \sum_{\mathbf{z}} \mathbb{P}(y \mid t=t, \mathbf{z}=z)\mathbb{P}(\mathbf{z}=z)$$

Front-door Criterion



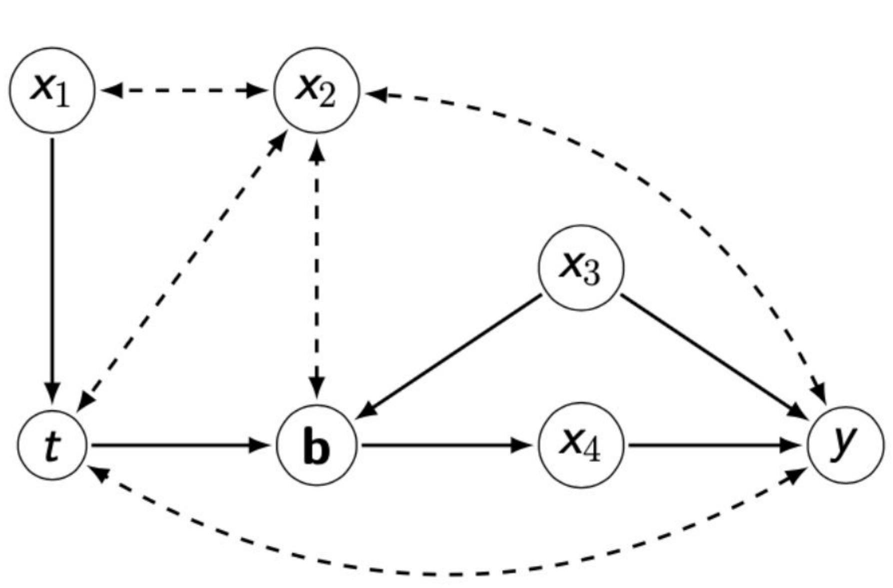
A set $\mathbf{z}$ satisfies the front-door criterion in $\mathcal{G}$ if

1. $\mathbf{z}$ blocks all directed paths from t to y in $\mathcal{G}$
2. all back-door paths between t and $\mathbf{z}$ in $\mathcal{G}$ are blocked
3. all back-door paths between $\mathbf{z}$ and y in $\mathcal{G}$ are blocked by t

$$\mathbb{P}(y \mid do(t=t)) = \sum_{\mathbf{z}} \Big(\sum_{t'} \mathbb{P}(y \mid t=t', \mathbf{z}=z)\mathbb{P}(t=t')\Big)\mathbb{P}(\mathbf{z}=z \mid t=t)$$

How much of the DAG do we need to know?

To find the causal effect when there is a confounder between t and y

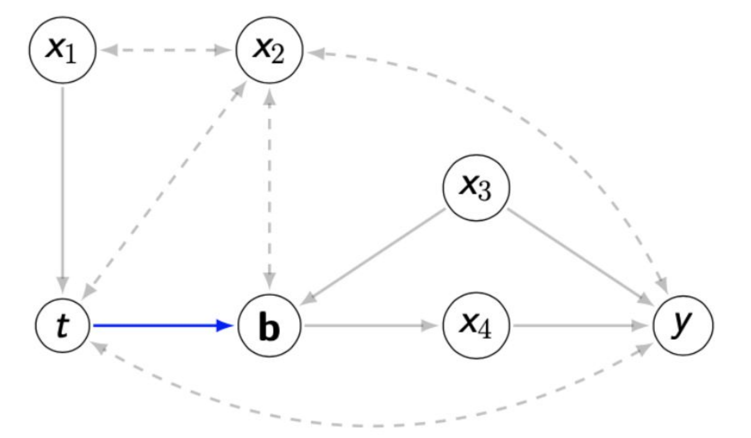


Assumptions

Semi-Markovian model

1. The outcome y is a descendant of the treatment t.
2. There is an unobserved confounder between the outcome y and the treatment t.

How to identify front-door-like sets using only partial structural information about post-treatment variables?



The knowledge of the children of the treatment is sufficient!

Causal Identifiability

- $\mathbf{b}$: the set of all children of t.
- Consider any subset of the remaining observed features i.e., $\mathbf{z} \subseteq \mathbf{x} \backslash \{\mathbf{b}\}$.
- $\exists\, \mathbf{z}$ such that $\mathbf{b} \perp y \mid t, \mathbf{z} \implies \mathbb{P}(y \mid do(t=t))$ is identifiable from observational data

$\mathbf{z} = (x_3, x_4)$ in $\mathcal{G}_{toy}$

Generalized front-door criterion

- If $\mathbf{z}$ satisfying $\mathbf{b} \perp y \mid t, \mathbf{z}$ can be decomposed into $\mathbf{z}^{(o)} \subseteq \mathbf{z}$ and $\mathbf{z}^{(i)} = \mathbf{z} \backslash \mathbf{z}^{(o)}$ such that

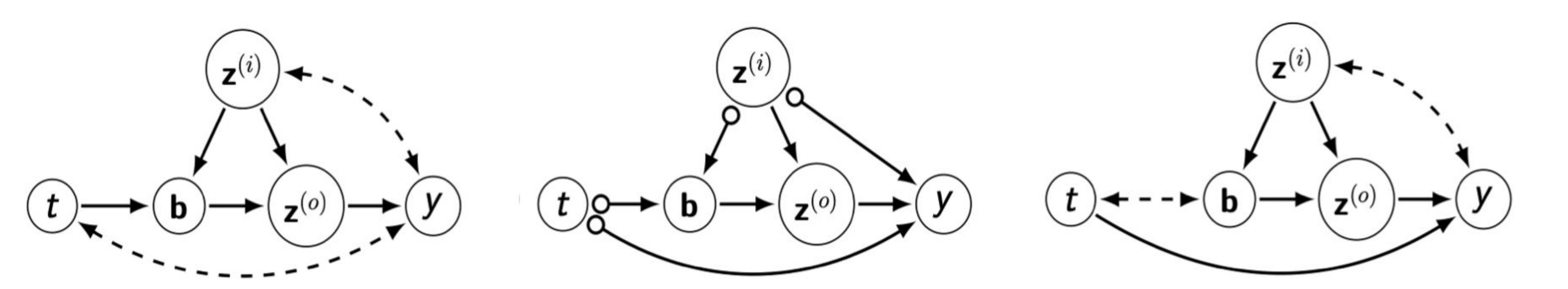
$$\mathbf{z}^{(i)} \perp t \qquad \text{and} \qquad \mathbf{z}^{(o)} \perp t \mid \mathbf{z}^{(i)}, \mathbf{b}$$

then, $\mathbf{z}$ and $\mathbf{s} \triangleq (\mathbf{b}, \mathbf{z}^{(i)})$ are generalized front-door sets, i.e.,

$$\mathbb{P}(y \mid do(t=t)) = \sum_{\mathbf{z}} \Big(\sum_{t'} \mathbb{P}(y \mid t=t', \mathbf{z}=z)\mathbb{P}(t=t')\Big)\mathbb{P}(\mathbf{z}=z \mid t=t)$$
$$= \sum_{\mathbf{s}} \Big(\sum_{t'} \mathbb{P}(y \mid t=t', \mathbf{s}=s)\mathbb{P}(t=t')\Big)\mathbb{P}(\mathbf{s}=s \mid t=t)$$

$\mathbf{z}^{(i)} = x_3$ and $\mathbf{z}^{(o)} = x_4$ in $\mathcal{G}_{toy}$

Relation to PAG-based algorithms



Graph $\mathcal{G}_1$ where our method applies

PAG corresponding to graphs $\mathcal{G}_1$ and $\mathcal{G}_2$

IDP algorithm fails here!

Graph $\mathcal{G}_2$ which is Markov equivalent to $\mathcal{G}_1$

Applicability to random graphs

- For p observed variables, let $v_1, \cdots, v_p$ denote a causal ordering.
- For $1 \leq i < j \leq p$, add $v_i \rightarrow v_j$ w.p. 0.5 if $j \leq 2d$ and w.p $d/(j-1)$ if $j > 2d$.
  - $\implies$ d is the expected in-degree of variables $v_{2d}, \cdots, v_p$.
- For $1 \leq i < j \leq p$, add $v_i \leftarrow - - \rightarrow v_j$ w.p. q/p.
  - $\implies$ q controls the number of unobserved variables.

Let $y = v_p$; t = any non-parent and non-grand parent ancestor of y; $\mathbf{b}$ = all children of t

	$p=10$			$p=15$		
	$d=2$	$d=3$	$d=4$	$d=2$	$d=3$	$d=4$
$q=0.0$	(43, 43)	(20, 20)	(21, 21)	(27, 26)	(9, 9)	(4, 2)
$q=0.5$	(23, 23)	(16, 16)	(7, 7)	(18, 17)	(4, 3)	(0, 0)
$q=1.0$	(6, 6)	(4, 4)	(5, 5)	(9, 9)	(10, 9)	(0, 0)

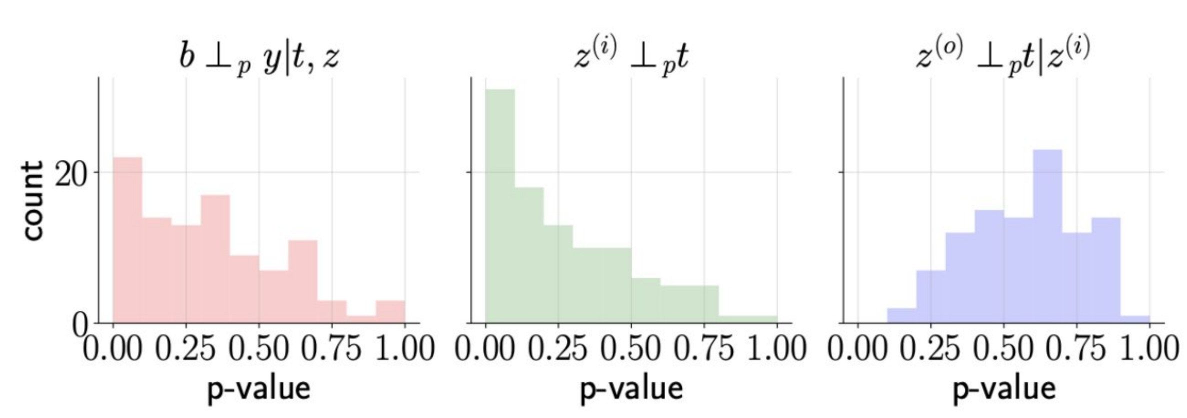
IDP gives 0 success out of 100 across for various $p, d,$ and q

German Credit Dataset — Fairness application

Compute the total effect of the sensitive attribute t on the outcome y

y = binary credit risk; t = applicant's age (binarized);

$\mathbf{b}$ = # of people financially dependent on the applicant, applicant's savings, applicant's job



Our method finds $\mathbf{z} = (\mathbf{z}^{(i)}, \mathbf{z}^{(o)})$ and results in

Total effect$_z = 0.0125 \pm 0.0011$, and Total effect$_s = 0.0105 \pm 0.0018$